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A NOTE ON THE NUMBER OF t-CORE PARTITIONS

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ABSTRACT. A partition of a positive integer n is a nonincreasing sequence of positive integers whose sum is n. A Ferrers graph represents a partition in the natural way. Fix a positive integer t. A partition of n is called a *t-core* partition of n if none of its hook numbers are multiples of t. Let $c_t(n)$ denote the number of t-core partitions of n. It has been conjectured that if $t \geq 4$, then $c_t(n) > 0$ for all $n \geq 0$. In [**7**], the author proved the conjecture for $t \geq 4$ even and for those t divisible by at least one of 5, 7, 9, or 11. Moreover if $t \geq 5$ is odd, then it was shown that $c_t(n) > 0$ for n sufficiently large. In this note we show that if $k \geq 2$, then $c_{3k}(n) > 0$ for all n using elementary arguments.

A partition of a positive integer n is a nonincreasing sequence of positive integers with sum n. Here we define a special class of partitions.

Definition 1. Let $t \geq 1$ be a positive integer. Any partition of n whose Ferrers graph have no hook numbers divisible by t is known as a *t-core partition* of n.

The hooks are important in the representation theory of finite symmetric groups and the theory of cranks associated with Ramanujan's congruences for the ordinary partition function [**3, 4, 5**].

If $t \geq 1$ and $n \geq 0$, then we define $c_t(n)$ to be the number of partitions of n that are t-core partitions. The arithmetic of $c_t(n)$ is studied in [**3, 4**]. The power series generating function for $c_t(n)$ is given by the infinite product:

$$\sum_{n=0}^{\infty} c_t(n)q^n = \prod_{n=1}^{\infty} \frac{(1-q^{tn})^t}{(1-q^n)}. \tag{1}$$

One easily verifies that $c_2(n)$ and $c_3(n)$ are zero infinitely often. Here

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