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TOPOLOGICAL NEARRINGS WHOSE ADDITIVE GROUPS ARE TORI

K.D. MAGILL, JR.

1. Introduction. A nearring is a triple $(N,+,*)$ where $(N,+)$ is a group, $(N,*)$ is a semigroup and $(x+y)*z=(x*z)+(y*z)$ for all $x,y,z\in N$. For information about the algebraic theory of nearrings, one may consult [**4, 8** and **9**]. If the binary operations $+$ and $*$ are continuous, then $(N,+,*)$ is a topological nearring. This paper was motivated by the following question, "Given a topological group $(G,+)$, exactly what are the continuous multiplications $*$ on G such that $(G,+,*)$ is a topological nearring?" The answer, it turns out, involves knowing just what the continuous functions are from G into the space of endomorphisms of G under the compact-open topology. We apply this general result, which is a topological version of a theorem of J.R. Clay [**2**] to the n-dimensional torus T^n and we are able to completely describe those multiplications $*$ so that $(T^n,+,*)$ is a topological nearring. One reason that the case for T^n follows so quickly is that there are, in a certain sense, few continuous maps from T^n into its space of endomorphisms. The case is far different, however, for the Euclidean n-groups. There are many continuous functions from R^n into its space of endomorphisms and, consequently, the operations $*$ for which $(R^n,+,*)$ is a topological nearring are much more abundant and varied. We will begin our investigation of continuous nearring multiplications on R^n in a subsequent paper. In this paper, after we derive the general result, we focus entirely on applications to the n-dimensional torus. The main results of the paper are in Section 2 where we derive the general result and then apply it to the n-dimensional torus in order to explicitly describe all the continuous multiplications $*$ on T^n such that $(T^n,+,*)$ is a topological nearring. After we describe these multiplications in Section 2, we derive a few corollaries and then we determine the ideals of each such nearring. In Section 3 we determine all the homomorphisms from one such nearring into another, and we describe the endomorphism semigroups and the automorphism groups

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