A REFLEXIVE SPACE WITH NORMAL STRUCTURE THAT ADMITS NO UCED NORM

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1. Introduction. A Banach space $X$ is said to have normal structure if every bounded convex subset $C$ of $X$ with positive diameter $d = \sup \{\|x - y\| : x, y \in C\}$ is contained in some ball with center in $C$ and radius strictly smaller than $d$. This property was introduced by Brodskii and Milman [2] and happened to be important in the fixed point theory for nonexpansive mappings.

It was proved in [6] and [3] that uniform convexity in every direction implies normal structure. An example was constructed in [4] of a reflexive space $Y$ without equivalent norm, uniformly convex in every direction, which answered a question in [3]. It is not difficult to see that the original norm of $Y$ does not have normal structure. However, we shall prove here that $Y$ admits an equivalent norm with normal structure. Since $Y$ gives the only known pattern for constructing reflexive spaces without equivalent UCED norms, the problem if every reflexive space admits an equivalent norm with normal structure remains open (see [1]). In fact, the main result of the present paper was stated in [5], but the proof was not correct because of a misunderstanding of the construction in [4]. Since this article is to be considered as a correction to [5], we shall use almost the same notation.

2. Notation and results. A Banach space $(Y, \| \cdot \|)$ is said to be uniformly convex in every direction if the conditions $x_n, y_n, z \in Y$, $\|x_n\| \to 1$, $\|y_n\| \to 1$, $\|(x_n + y_n)/2\| \to 1$ and $x_n - y_n = \lambda_n z$, $\lambda_n$ reals, imply that $\|x_n - y_n\| \to 0$.

Following [5], for $Z = (\mathbb{R}^n, \| \cdot \|)$ with symmetric norm $\| \cdot \|$ the $Z$-direct sum of the normed spaces $X_1, \ldots, X_n$ is its product space with norm $\|(x_1, \ldots, x_n)\| = (\|x_1\|, \ldots, \|x_n\|)$. A normed space $X$ is said to have the sum-property if each $Z$-direct sum of finitely many copies of $X$ has