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A REFLEXIVE SPACE WITH NORMAL STRUCTURE THAT ADMITS NO UCED NORM

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1. Introduction. A Banach space X is said to have normal structure if every bounded convex subset C of X with positive diameter $d = \sup\{||x-y|| : x, y \in C\}$ is contained in some ball with center in C and radius strictly smaller than d. This property was introduced by Brodskii and Milman [**2**] and happened to be important in the fixed point theory for nonexpansive mappings.

It was proved in [**6**] and [**3**] that uniform convexity in every direction implies normal structure. An example was constructed in [**4**] of a reflexive space Y without equivalent norm, uniformly convex in every direction, which answered a question in [**3**]. It is not difficult to see that the original norm of Y does not have normal structure. However, we shall prove here that Y admits an equivalent norm with normal structure. Since Y gives the only known pattern for constructing reflexive spaces without equivalent UCED norms, the problem if every reflexive space admits an equivalent norm with normal structure remains open (see [**1**]). In fact, the main result of the present paper was stated in [**5**], but the proof was not correct because of a misunderstanding of the construction in [**4**]. Since this article is to be considered as a correction to [**5**], we shall use almost the same notation.

2. Notation and results. A Banach space $(Y, ||\cdot||)$ is said to be uniformly convex in every direction if the conditions $x_n, y_n,\ z \in Y$, $||x_n|| \to 1$, $||y_n|| \to 1$, $||(x_n+y_n)/2|| \to 1$ and $x_n - y_n = \lambda_n z$, λ_n reals, imply that $||x_n - y_n|| \to 0$.

Following [**5**], for $Z = (\mathbf{R}^n, |\cdot|)$ with symmetric norm $|\cdot|$ the Z-direct sum of the normed spaces $X_1, \dots, X_n$ is its product space with norm $||(x_1, \dots, x_n)|| = |(||x_1||, \dots, ||x_n||)|$. A normed space X is said to have the sum-property if each Z-direct sum of finitely many copies of X has

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