ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 3, Summer 1995

NONPROBABILISTIC COMPUTATION OF THE POISSON BOUNDARY FOR AN ÉTALÉE MEASURE ON A SEMI-SIMPLE LIE GROUP

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0. Introduction. In [**6**], Furstenberg proved the existence of a boundary representation for μ-harmonic functions on T, a connected semi-simple Lie group with finite center and no compact factors. In the same paper he also computed the boundary in the case μ was absolutely continuous with respect to Haar measure on T. In proving both these results he used probability theory (the Martingale theorem). In [**1**] Azencott generalized these results to the case of an étalée measure again using probability theory.

In [**2**] it was shown how to construct the boundary using nonprobabilistic techniques. These results were developed further in [**3**] applied to μ and T as in Furstenberg's situation with the additional hypothesis that μ was supported on T. The boundary was shown to be a compact homogeneous space of T.

In this paper we prove Azencott's generalization of Furstenberg's result (see Theorem 2.7) for the case of μ, a spread out étalée measure on T a semi-simple, connected Lie group, with finite center and no compact factors. The techniques we use are those developed in [**2**] and [**3**] and are nonprobabilistic in nature.

1. Review of basic concepts. In this section we review the basic definitions and constructions involved in the Poisson boundary (B, T, ω). For a more detailed discussion, see [**2**] and [**3**]. Let T be a locally compact Hausdorff topological group. The right uniformly continuous functions on T which are bounded form a Banach algebra, $\mathbf{R}$, with respect to pointwise multiplication and the supremum norm. Let P be the Gelfand space of $\mathbf{R}$. Then (P, T) is a flow and P has a semi-group structure such that if $p \in P$, then $l_p : P \to P : q \to p \cdot q$ is continuous, and if $t \in T$, then $r_t : P \to P : q \to q \cdot t$ is continuous.

Received by the editors on July 15, 1993.