ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 3, Summer 1995

ON THE UNIQUENESS OF THE POSITIVE SOLUTION OF A SINGULARLY PERTURBED PROBLEM

E.N. DANCER

0. Introduction. A number of authors have considered the existence of multiple positive solutions of

$$\begin{aligned} -\varepsilon\Delta u &= u^p - u \quad \text{in } D, \\ u &= 0 \quad \text{on } \partial D \end{aligned} \tag{1}$$

if ε is small, D is suitably complicated in R^n and

$$1 < p < (n-2)^{-1}(n+2).$$

See [**1, 20, 24** and **25**]. (Some of these consider the Neumann problem.) Here we consider the opposite situation and show that, if D has n distinct symmetries and some other properties (for example some form of generalized ellipsoid) and if $1 < p < (n-2)^{-1}(n+2)$, then the positive solution is unique for small positive ε. This provides an interesting contrast with the results above. Note that the results in [**3**] suggest that some strong geometric conditions on D are necessary for this result to be true. We actually discuss rather more general nonlinearities. Note that the behavior of the positive solutions for small ε is quite different from the cases in [**5**].

We also make some remarks on the case of large ε and the very different behavior of the Neumann problem. The very different behavior of the problem under different boundary conditions is another source of interest in the problem.

1. The main result. In this section we prove the main result. We consider a domain $D \subseteq R^n$ such that $0 \in D$, D has C^3 boundary,

Received by the editors on August 10, 1993, and in revised form on March 28, 1994.
This research was partially supported by the Australian Research Council.