

GROUPS OF PIECEWISE-LINEAR HOMEOMORPHISMS WITH IRRATIONAL SLOPES

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ABSTRACT. Let F be the group of piecewise-linear homeomorphisms of the unit interval. F has many interesting countable discrete subgroups, some of which have cohomological finiteness properties. Many subgroups of piecewise-linear homeomorphisms with irrational slopes and irrational singularities are finitely generated, finitely presented and are of type FP_∞ . This is shown by constructing contractible posets upon which the various subgroups act and then by understanding the complexity of the classifying space of the poset, which is an Eilenberg-MacLane space for the subgroup.

1. Introduction. Groups of piecewise-linear homeomorphisms have proven to be interesting examples of countable groups. The first such group was used in the construction of an early example of a finitely-presented infinite simple group by Thompson [6]. Brown and Geoghegan [3] showed later that it has a subgroup which is a finitely-presented, infinite-dimensional, torsion-free group. They also showed that subgroup to be of type FP_∞ by building a $K(G, 1)$ complex for the group which had only finitely many cells (in fact, two) in each dimension. The group that Brown and Geoghegan studied can be described as the subgroup of the group of all piecewise-linear homeomorphisms of the interval $[0, 1]$ where every element of the group has only finitely many singularities, each singularity lies in the dyadic numbers, $\mathbf{Z}[1/2]$, and the slope of the homeomorphisms away from the singularities lie in $\{2^i, i \in \mathbf{Z}\}$. Brown [2] later studied generalizations of this group where the singularity set and slope set are respectively $\mathbf{Z}[1/p]$ and $\{p^i, i \in \mathbf{Z}\}$. He showed that all these groups are also of type FP_∞ . Stein [5] studied such groups for singularity set $\mathbf{Z}[1/p_1, 1/p_2, \dots, 1/p_k]$ with slope group $\{p_1^{i_1} p_2^{i_2} \cdots p_k^{i_k}, i_j \in \mathbf{Z}\}$ and showed that all these groups are of type FP_∞ .

I study another class of groups of piecewise-linear homeomorphisms where the singularities can be at irrational points, for instance in $\mathbf{Z}[\sqrt{2}]$. Many of these groups are also of type FP_∞ .

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