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WEIGHTED L^2-MULTIPLIERS

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ABSTRACT. In this paper we give a simple proof of the characterization of an L^2 multiplier with weight $|x|^{2k}$, $k \in \mathbf{N}$.

1. Introduction. Let $f \mapsto \hat{f}$ be the Fourier transform, $f \mapsto \check{f}$ the inverse Fourier transform, and m a bounded measurable function on $\mathbf{R}$. We say that m is a *multiplier* for $L^p(\mathbf{R})$, $1 \le p \le \infty$, if $f \in L^2 \cap L^p$ implies $(m\hat{f})\check{}$ is in L^p and satisfies

$$||(m\hat{f})\check{}\,||_p \le C_p ||f||_p \quad \text{with } C_p \text{ independent of } f.$$

For $\alpha \ge 0$, we express $L^2(x^{2\alpha})$ the collection of f with $||f||^2_{L^2(x^{2\alpha})} = \int_{-\infty}^{\infty} |x^\alpha f(x)|^2\,dx < \infty$, and $\mathcal{S}_{00}(\mathbf{R}) = \{f$ in the Schwartz class $\mathcal{S}(\mathbf{R})$: $\hat{f}$ has compact support not including the origin$\}$. For $f \in \mathcal{S}_{00}$, it is easy to check that $xf \in \mathcal{S}_{00}$ and $\hat{f}^{(k)}$ vanishes in a neighborhood of the origin for all $k \in \mathbf{N}$. Thus we have $\hat{f}^{(k)}(x) = \int_0^x \hat{f}^{(k+1)}(t)\,dt$ for all $k \in \mathbf{N}$. Furthermore, it is well known that $\mathcal{S}_{00}$ is dense in $L^2(x^{2\alpha})$ (see [**4**]).

Hörmander [**1**] gave a sufficient condition for multipliers in 1960. Kurtz and Wheeden [**2**] proved a weighted version of the Hörmander multiplier theorem. Both gave sufficient conditions for multipliers, but not necessary conditions. Muckenhoupt, Wheeden, and Young [**4**] provided sufficient and necessary conditions for L^2 multipliers with power weight $|x|^{2\alpha}$, $\alpha \in \mathbf{R}$. In this paper we use the principle of mathematical induction to give a simple proof of the characterization of an L^2 multiplier with weight $|x|^{2k}$, $k \in \mathbf{N}$. Finally, we mention that C will be used to denote a constant which may vary from line to line.

We recall that Hardy's inequality with weights is stated as following.

Theorem 1 [**3**]. *If $1 \le p \le \infty$, there is a finite constant C for which*

$$\left(\int_0^\infty \left|U(x)\int_0^x f(t)\,dt\right|^p dx\right)^{1/p} \le C\left(\int_0^\infty |V(x)f(x)|^p\,dx\right)^{1/p} \tag{1.1}$$

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