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FUNCTION THEORIES FOR THE YUKAWA AND HELMHOLTZ EQUATIONS

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ABSTRACT. Transmutations operating on heat polynomials and associated heat functions are employed to develop function theories for the Yukawa and Helmholtz equations. The special functions developed by means of these transmutations are studied, including series and integral representation theorems for solutions corresponding to analytic and entire data.

1. Introduction. It is well known [**1**] that the linear second order partial differential equation in two independent variables

$$au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + fu = g(x, y) \tag{1.1}$$

with constant coefficients can (except for one degenerate case) be transformed by changes of variables to the heat, wave, Laplace, Helmholtz or Yukawa equation. The best known function theory associated with these equations, in the case of Laplace's equation, is the analytic function theory in one complex variable. P.C. Rosenbloom and D.V. Widder [**13**] have developed a function theory for the heat equation based on the heat polynomials and associated heat functions. The authors [**5**] have shown that there is an analogous function theory for the wave equation related to these through transmutation operators. In the present paper, we show that there are analogous function theories for the Yukawa and Helmholtz equations. R.J. Duffin [**8**] has presented a function theory of the Yukawa equation from the point of view of the pseudoanalytic function theory of Bers-Vekua [**2**]. Also, see [**9**]. Our approach is through transmutation operators [**3, 7**] and is essentially independent of the Bers-Vekua theory. There are also analogous function theories for certain singular elliptic and hyperbolic differential equations [**6**].

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