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ε-SPACES

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ABSTRACT. Consider a Tychonoff space, X, and the lattice-ordered group $C(X)$ of real-valued continuous functions. Within a certain category of l-groups, the "epicomplete epireflection" $\varepsilon C(X)$ of $C(X)$ looks enough like the l-group $B(X)$ of Baire functions on X to present the question: For what X is $\varepsilon C(X) = B(X)$? That equality means just that each homomorphism from $C(X)$ to an epicomplete target lifts to a homomorphism of $B(X)$ and is, we show, equivalent to this condition on the placement of X in its Stone-Čech compactification βX: If E is a Baire set of βX which misses X, then there are zero-sets $Z_1, Z_2, \dots$ of βX for which $E \subseteq \cup_n Z_n \subseteq \beta X - X$. We call such an X an "ε-space" and examine these spaces, rather inconclusively.

Algebra to topology. In the first two sections we present a synopsis of the theory in [**1, 2, 3**] to motivate the question, "$\varepsilon C(X) = B(X)$?" and to make the topological reduction described in the abstract. The reader who finds the definition of ε-spaces in the abstract sufficiently compelling can, for the most part, just skip to Section 3.

1. Epicompleteness. $\mathcal{W}$ is the category of Archimedean l-groups with distinguished weak order unit and morphisms the l-homomorphisms which preserve unit. Each $C(X)$, with unit the constant function 1, is an object of $\mathcal{W}$, the $\mathcal{W}$-morphisms between $C(X)$'s are exactly the homomorphisms described in Chapter 10 of [**7**] and, in many other ways, the category $\mathcal{W}$ generalizes (significantly) the theory of $C(X)$ in [**7**]; see, e.g., [**8**]. The discussion of this section takes place "in $\mathcal{W}$."

An epimorphism (or just "epic") is a homomorphism $\alpha : A \to B$ for which $\gamma\alpha = \delta\alpha$ (with γ, δ homomorphisms) implies $\gamma = \delta$. [**1**] contains an explicit description of the epics, but we can skip this.

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