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ON THE TRUNCATION OF FUNCTIONS IN LORENTZ AND MARCINKIEWICZ SPACES

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ABSTRACT. Given a measurable function x on $[0,1]$, we study the family $Q(x)$ of all quasi-concave functions ψ such that $||x_h||_{M(\psi)} = o(||x_h||_{\Lambda(\psi)})$ as $h \to \infty$, where x_h denotes the truncation of x at height h. We show, in particular, that $Q(x)$ is nonempty if and only if $x \in L_1 \backslash L_\infty$.

Recall that a Banach space E of measurable functions on $[0,1]$ is called *symmetric space* or *rearrangement invariant* (r.i.) *space* if the following holds:

(a) from $|x(t)| \leq |y(t)|$ and $y \in E$ it follows that $x \in E$ and $||x||_E \leq ||y||_E$;

(b) if x is equi-measurable to $y \in E$, then $x \in E$ and $||x||_E = ||y||_E$.

Denote by χ_e the characteristic function of a measurable set $e \subseteq [0,1]$. By (b), the norm $||\chi_e||_E$ depends only then on the measure μe of e. Consequently, the function $\varphi_E : [0,1] \to [0,\infty)$ given by $\varphi_E(\mu e) = ||\chi_e||_E$ (the so-called *fundamental function* of E) is well-defined.

Examples of r.i. spaces are the classical Lebesgue, Orlicz, Lorentz and Marcinkiewicz spaces. Denote by Ω the set of all quasi-concave functions $\psi : [0,1] \to [0,\infty)$, i.e., $\psi(0) = 0$, and both functions $t \mapsto \psi(t)$ and $t \mapsto t/\psi(t)$ are increasing. Given $\psi \in \Omega$, let

$$||x||_{\Lambda(\psi)} = \int_0^1 x^*(t)\, d\psi(t) \tag{1}$$

and

$$||x||_{M(\psi)} = \sup_{0<\tau\leq 1} \frac{\psi(\tau)}{\tau} \int_0^\tau x^*(t)\, dt \tag{2}$$

where $x^*(t)$ denotes the decreasing rearrangement of $|x(t)|$. The space $\Lambda(\psi)$ defined by the norm (1) is usually called *Lorentz space*, the space

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