GENERALIZATION OF AN INEQUALITY FOR NONDECREASING SEQUENCES

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In 1981, A. Meir [4] proved the following theorem for nondecreasing sequences:

Theorem A. Let $a_0, a_1, \ldots, a_{n-1}$ and p_1, p_2, \ldots, p_n be nonnegative real numbers satisfying

$$0 = a_0 \le a_1 \le a_2 \le \dots \le a_{n-1},$$

$$a_i - a_{i-1} \le p_i, \quad i = 1, \dots, n-1,$$

and

$$(1) p_1 \le p_2 \le \dots \le p_n.$$

If r and s are real numbers with $r \geq 1$ and $s \geq 2r + 1$, then

(2)
$$\left[(s+1) \sum_{i=1}^{n-1} a_i^s \frac{p_i + p_{i+1}}{2} \right]^{1/(s+1)} \le \left[(r+1) \sum_{i=1}^{n-1} a_i^r \frac{p_i + p_{i+1}}{2} \right]^{1/(r+1)}.$$

In 1986, G.V. Milovanović and I.Ž. Milovanović [5] presented an interesting refinement of Theorem A. Their result states:

Theorem B. If the numbers a_i , i = 0, 1, ..., n-1, p_i , i = 1, 2, ..., n,

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