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ALMOST FLAT ABELIAN GROUPS

ULRICH ALBRECHT AND PAT GOETERS

1. Introduction. While investigating the relationship between group-theoretic properties of an abelian group A and ring-theoretic properties of its endomorphism ring, $E(A)$, it often becomes necessary to impose restrictions on A to avoid obvious counter-examples. This is frequently done by considering abelian groups A which, as a left $E(A)$-module, belong to a given class $\mathcal{C}$ of modules. Perhaps the most frequently used choices for $\mathcal{C}$ are the classes of cyclic, finitely generated, projective, or flat left $E(A)$-modules. Although all these choices yield interesting results, only the condition that A is a flat $E(A)$-module does not impose immediate restrictions on the ring-structure of $E(A)$, see, e.g., [**2**] and [**4**]. R.S. Pierce emphasized the importance of the flatness condition on A in a talk, which he gave in 1989 at the University of Connecticut, and raised several questions concerning these groups [**8**]: Is the class of torsion-free groups which are flat as $E(A)$-module closed with respect to quasi- or near-isomorphism? What is the relationship between the flatness of A over $E(A)$, and the flatness of $\mathbf{Q}A = \mathbf{Q} \otimes_{\mathbf{Z}} A$ as a $\mathbf{Q} \otimes_{\mathbf{Z}} E(A)$-module?

It is the goal of this paper to give answers to Pierce's questions. Example 2.9 shows that the class of abelian groups A which are flat as $E(A)$-modules need not be closed under quasi-isomorphism. This observation leads to the introduction of a new class of abelian groups. We say that A is *almost flat as an* $E(A)$*-module* if $\operatorname{Tor}^1_{E(A)}(M, A)$ is a bounded abelian group for all right $E(A)$-modules M. Theorem 2.4 and Corollary 2.5 establish that the class of torsion-free abelian groups, which are almost flat over their endomorphism ring, is closed under quasi-isomorphism. Furthermore, the arguments used in the proof of Corollary 2.5 can be adopted to show that the class of torsion-free abelian groups, which are flat as $E(A)$-modules, is closed under near-isomorphism (Corollary 2.7).

It remains to investigate how the class of abelian groups A, which are

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