ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 4, Fall 1995

UNIQUE CONTINUATION OF WEAKLY CONFORMAL MAPPINGS BETWEEN RIEMANNIAN MANIFOLDS

YIFEI PAN

ABSTRACT. In this note we show that weakly conformal mappings between two Riemannian manifolds satisfy the strong unique continuation property in a sense that if the conformal factor of a weakly conformal mapping vanishes to infinite order at a point, then it is a constant map.

1. Introduction. Let (M, g) and (N, h) be Riemannian manifolds. We assume in this paper that M and N are connected. A smooth map $\phi : M \to N$ is said to be weakly conformal if $\phi^* h = \lambda g$ with $\lambda \geq 0$ and conformal if $\lambda > 0$ on M, where the scalar function λ is a so-called conformal factor of ϕ. It is well-known that solutions of elliptic partial differential equations satisfy the strong unique continuation property in a sense that if they vanish to infinite order at a point then they are identically zero. In this note we show that weakly conformal mappings between Riemannian manifolds also satisfy the strong unique continuation property. More precisely,

Theorem. *Let $\phi : M \to N$ be a weakly conformal mapping between two Riemannian manifolds of equal dimension. Then ϕ satisfies the strong unique continuation property; that is, given a point p in M, if the conformal factor λ of ϕ vanishes to infinite order at p, then $\phi \equiv \phi(p)$ on M.*

Corollary. *If two weakly conformal maps agree in a neighborhood of a point, then they are equal.*

Corollary. *Let $\phi : (M, g) \to (N, h)$ be a nonconstant weakly conformal mapping between two Riemannian manifolds of equal dimension. Then $\phi^*(h)$, the pullback of metric h on N, induces a metric on M ex-*

Received by the editors on November 22, 1993.