ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 4, Fall 1995

# AN OSCILLATION CRITERION OF ALMOST-PERIODIC STURM-LIOUVILLE EQUATIONS

HORNG JAAN LI AND CHEH-CHIH YEH

ABSTRACT. The class $\Omega \subset L^1_{\text{loc}}(\mathbf{R})$ of Besicovitch almost-periodic functions is the closure of the set of all finite trigonometric polynomials with the Besicovitch seminorm. Consider the half-linear second order differential equation

$$(E) \qquad \frac{d}{dt}\phi(u'(t)) - \lambda c(t)\phi(u(t)) = 0,$$

where $\phi(s) = |s|^{p-2}s$ with $p > 1$ a fixed number and $c(t) \in \Omega$. We show that if $M\{c\} := \lim_{t\to\infty}(1/t)\int_0^t c(s+\alpha)\,ds = 0$ and $M\{|c|\} > 0$, then (E) is oscillatory at $+\infty$ and $-\infty$ for every $\lambda \in \mathbf{R} - \{0\}$.

**1. Introduction.** Let $\mathbf{R}$ denote the real line. The class $\Omega \subset L^1_{\text{loc}}(\mathbf{R})$ of Besicovitch almost-periodic functions is the closure of the set of all finite trigonometric polynomials with the Besicovitch seminorm $||\cdot||_B$:

$$||c||_B = \limsup_{t\to\infty} \frac{1}{2t}\int_{-t}^{t} |c(s)|\,ds,$$

where $c \in \Omega$. The mean value, $M\{c\}$, of $c \in \Omega$, always exists, is finite and is uniform with respect to $\alpha$ for $\alpha \in \mathbf{R}$, where

$$M\{c\} = \lim_{t\to\infty} \frac{1}{t}\int_{t_0}^{t} c(s+\alpha)\,ds,$$

for some $t_0 \geq 0$ (see [**1**] and [**3**] for details).

---

Received by the editors on August 25, 1993, and in revised form on January 10, 1995.
1980 *Mathematics Subject Classification* (1985 Revision). Primary 34C10, 34C15.
*Key words and phrases.* Besicovitch almost-periodic function,half-linear second order, oscillatory, Euler-Lagrange equation, Levin's comparison theorem.
This research was supported by the National Science Council of the Republic of China.