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QUOTIENTS OF G-STABLE CLOSED SUBSCHEMES, CARTESIAN DIAGRAMS, AND CLOSED IMMERSIONS

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ABSTRACT. Let T be a separated scheme of finite type over an algebraically closed field k, G a finite group acting on T, and $i : S \hookrightarrow T$ the inclusion of a G-stable closed subscheme (i.e., for all $g \in G$, the scheme-theoretic image of S under the composite map $S \overset{i}{\hookrightarrow} T \overset{g_T}{\to} T$ is equal to S). It then follows that S inherits a G-action. If the quotient T/G exists, then so does the quotient S/G; the universal property of the quotient gives rise to a map $i/G : S/G \to T/G$. We ask two questions: Is the commutative square formed by the maps i, i/G, and the quotient maps π_S, π_T, cartesian? Is the map i/G a closed immersion? In case G acts freely on T, we show that both answers are "yes." On the other hand, suppose $T = X^n \times X$ for X a quasiprojective variety, G is a symmetric group on n letters acting by permuting the factors of X^n, and S is a reduced closed subscheme of T supported on the locus whose k-points are all $((t_1, \dots, t_n), t)$ such that $t = t_j$ for some j, $1 \leq j \leq n$. Then i/G is a closed immersion, but the square is not in general cartesian (but is so when X is a nonsingular curve). This corrects an error in the paper, "The Secant Bundle of a Projective Variety," by R.L.E. Schwarzenberger [**11**].

1. Introduction. We fix an algebraically closed field k. Let T be a *k-scheme*, that is, a separated scheme of finite type over k, and let G be a finite group acting on T; for $g \in G$, we write $g_T : T \to T$ for the associated automorphism of T. Let $i : S \hookrightarrow T$ be the inclusion of a G-stable closed subscheme (i.e., for all $g \in G$, the scheme-theoretic image of S under the composite map $S \overset{i}{\hookrightarrow} T \overset{g_T}{\to} T$ is equal to S). It then follows that S inherits a G-action such that i is a G-morphism (for all $g \in G$, $i \circ g_S = g_T \circ i$). If the quotient T/G exists, then so does the quotient S/G; the universal property of the quotient then gives rise to the map i/G in the following commutative square (in which the vertical

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