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NONRESONANCE CONDITIONS ON THE POTENTIAL WITH RESPECT TO THE FUČIK SPECTRUM FOR THE PERIODIC BOUNDARY VALUE PROBLEM

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ABSTRACT. The existence of periodic solutions to a class of second order nonlinear ordinary differential equations is established under some rather mild restrictions on the behavior of the primitive of the nonlinearity with respect to the Fučik spectrum of the periodic problem.

1. Introduction. In this paper we study the solvability of the periodic boundary value problem

$$u'' + g(u) = h(t), \tag{1.1}$$

$$u(0) = u(2\pi), \qquad u'(0) = u'(2\pi), \tag{1.2}$$

where $g : \mathbf{R} \to \mathbf{R}$ is continuous and $h : [0, 2\pi] \to \mathbf{R}$ is Lebesgue integrable. The conditions we consider relate the asymptotic behavior of $g(s)$ and of its primitive $G(s) = \int_{[0,s]} g(\xi)\, d\xi$, with the Dancer-Fučik spectrum of the positively homogeneous problem

$$u'' + \mu u^+ - \nu u^- = 0, \tag{1.3}$$

subject to the boundary conditions (1.2). We recall that the Dancer-Fučik spectrum $\mathcal{S}$ (cf. [**4, 12**]) is made by all pairs $(\mu, \nu) \in \mathbf{R}^2$ such that (1.3)–(1.2) has nontrivial solutions. Precisely, it can be expressed as

$$\mathcal{S} = \bigcup_{m\in\mathbf{N}} \mathcal{C}_m,$$

where

$$\mathcal{C}_0 = \{(\mu, \nu) : \mu\nu = 0\}$$

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