

NONLINEAR OSCILLATION OF FIRST ORDER DELAY DIFFERENTIAL EQUATIONS

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ABSTRACT. First order delay differential equations with forcing term and the related differential equations are studied and sufficient conditions are derived for all solutions to be oscillatory.

1. Introduction. Oscillation properties of first order functional differential equations has been investigated by many authors. We refer the reader to Bainov and Mishev [1], Györi and Ladas [3], Ladde, Lakshmikantham and Zhang [4] and the references cited therein. In particular, the oscillation of functional differential equations with forcing term was studied by Onose [5, 6] and Tomaras [7]. However, it seems that very little is known about the sufficient conditions which imply that all solutions of certain nonlinear functional differential equation with forcing term are oscillatory.

The objective of this paper is to establish oscillation criteria for the delay differential equation

$$(1) \quad y'(t) + p(t)f(y(\sigma(t))) = q(t), \quad t > t_0,$$

and the related differential equation

$$(2) \quad y'(t) + a(t)y(t) + \sum_{i=1}^k b_i(t)y(\rho_i(t)) + p(t)f(y(\sigma(t))) = q(t), \quad t > t_0,$$

where t_0 is a positive number. In what follows, by a *solution* of (1) or (2), we mean a function $y(t) \in C([t_{-1}, \infty); \mathbf{R}^1) \cap C^1([t_0, \infty); \mathbf{R}^1)$ which satisfies (1) or (2) for all $t > t_0$, where

$$t_{-1} = \begin{cases} \inf_{t \geq t_0} \sigma(t) & \text{in the case of (1)} \\ \min\{\inf_{t \geq t_0} \sigma(t), \min_{1 \leq i \leq k} \inf_{t \geq t_0} \rho_i(t)\} & \text{in the case of (2)}. \end{cases}$$

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