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MOMENT MAPS FOR TORUS ACTIONS

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ABSTRACT. A surface of revolution in $\mathbf{R}^3$ has a natural symplectic S^1 action. A moment map for this action is constructed. Also, a converse to a theorem of Marsden and Weinstein on the existence of moment maps for torus actions is derived for compact surfaces.

1. Introduction. This paper examines when a moment map exists for a symplectic torus action on a compact manifold. This is an important question because these actions are useful tools in mathematical physics and in algebraic geometry (where the compact case is of more interest; see [**3**]). Most of the results apply to compact surfaces (2-manifolds); in fact, a lot of insight is gained by considering the rather simple case of the obvious S^1 action on a surface of revolution in $\mathbf{R}^3$. The most important result is a converse (in the case of a compact surface M) to a long-known result of Marsden and Weinstein [**6**] that a moment map for a torus action on M exists if $H^1(M, \mathbf{Q}) = 0$.

All manifolds are assumed differentiable, connected, compact and orientable. Singular cohomology is denoted H^i, while de Rham cohomology is denoted H^i_{dR}. G is an abelian Lie group, usually a torus (i.e., a product of S^1's), with Lie algebra $\underline{g}$.

Here are the basic definitions (to fix notation), mostly following [**1**]. A closed nondegenerate 2-form ω on the manifold M is called a *symplectic structure* on M. A trivial consequence of this definition is that every orientable 2-manifold has a symplectic structure given by the area form dA. (In fact, if M is a compact surface, then $H^2_{dR}(M, \mathbf{Q})$ has rank 1, so ω is a multiple of dA modulo exact forms.)

When G acts on a symplectic manifold (M, ω) we will consider each $g \in G$ as a diffeomorphism $g : M \to M$ and simply write gm instead of $g(m)$. The set $G.m = \{gm : g \in G\}$ is called the *orbit* through m. A G-action on M is *symplectic* if for all $g \in G$ we have $g^*(\omega) = \omega$. For each

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