ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 1, Winter 1996

ROBINSON'S THEOREM ON ASYMMETRIC DIOPHANTINE APPROXIMATION

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ABSTRACT. Let x be an irrational number. In this note we give two functions $A(K)$ and $B(K)$ defined on positive integers, for which the asymmetric approximation inequality $-1/(A(K)q^2) < x-p/q < 1/(B(K)q^2)$ has infinitely many rational solutions p/q. This result improves Robinson's classical asymmetric inequality found in 1947.

1. Introduction. In 1891, Hurwitz [**3**] proved the fundamental theorem on Diophantine approximation, which asserts that for any irrational number x, there are infinitely many rational numbers p/q such that $|x-p/q| < 1/(\sqrt{5}q^2)$. This inequality involves absolute value and is called symmetric approximation.

In 1945, Segre [**10**] initiated the study of asymmetric approximation. He proved that for any irrational numbers x and a given positive real number τ independent of x, there are infinitely many rational numbers p/q such that $-1/(\sqrt{1+4\tau}q^2) < x-p/q < \tau/(\sqrt{1+4\tau}q^2)$. Segre's result has been extensively investigated. See Mahler [**7**], Le Veque [**6**], Kopetzky and Schnitzer [**4, 5**], Prasad and Prasad [**8**], Szüsz [**11**] and Tong [**12–16**].

Right after Segre's discovery, Robinson [**9**] pointed out another direction of asymmetric approximation in 1947. He proved the following theorem:

Theorem 1. *Let x be an irrational number. Then for any given positive real number ε, there are infinitely many rational numbers p/q such that*

$$-1/((\sqrt{5}-\varepsilon)q^2) < x-p/q < 1/((1+\sqrt{5})q^2). \tag{1}$$

Received by the editors on January 8, 1994, and in revised form on August 18, 1994.
AMS (1990) *Mathematical Subject Classification.* Primary 11J04, 11A55.