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C^*-ALGEBRAS GENERATED BY COMMUTING ISOMETRIES

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ABSTRACT. C^*-algebras generated by commuting isometries are analyzed. It is shown that if a C^*-algebra is generated by a semigroup of commuting isometries whose range projections commute, then the C^*-algebra is nuclear. Not all C^*-algebras generated by commuting isometries are nuclear—the universal C^*-algebra generated by a commuting pair of isometries is shown to be nonnuclear.

1. Introduction. If G is a group, then—as is well known—its unitary representations correspond to the representations of the (full) group C^*-algebra $C^*(G)$. Thus, the algebras $C^*(G)$ can be used to reduce the representation theory of groups to that of a special case of the representation theory of C^*-algebras. Of course, the algebras $C^*(G)$ are important in their own right also, since they—and the corresponding reduced group C^*-algebras $C^*_{\rm red}(G)$—provide interesting examples in the theory of C^*-algebras. Indeed, the study of group C^*-algebras has played a significant role in the development of the general theory of C^*-algebras.

In analogy with the group case, one can associate with each cancellative semigroup M a C^*-algebra $C^*(M)$ that reflects the isometric representation theory of M (that is, the representations of M by isometries on Hilbert spaces). An early study of the algebras $C^*(M)$ was undertaken by R.G. Douglas in the special case that M is the positive cone of a subgroup of the additive group $\mathbf{R}$, see [**4**]. The more general analysis of C^*-algebras generated by commuting isometries undertaken by C.A. Berger, L.A. Coburn and A. Lebow in [**1**] is particularly relevant to the considerations of this paper. More recently, the author has made a detailed study of semigroup algebras for the case in which the semigroup M is the positive cone of an ordered group [**9, 10, 11**]. In this case $C^*(M)$ is primitive and nuclear and has many other nice properties, some of which are discussed below.

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