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TORSION OF DIFFERENTIALS OF AFFINE QUASI-HOMOGENEOUS HYPERSURFACES

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ABSTRACT. In this paper we prove that the torsion modules of the module of Kaehler differentials of affine hypersurfaces defined by a reduced quasi-homogeneous polynomial with an isolated singularity at the origin are cyclic. We give explicit expressions for generators. Moreover, we exhibit an isomorphism between the torsion submodule of $\Omega^{N-1}_{A/K}$ and $\Omega^{N}_{A/K}$ for such hypersurfaces. $A-D-E$ singularities provide examples of such hypersurfaces.

0. Introduction. Let K be a field of characteristic zero. We consider reduced affine quasi-homogeneous hypersurfaces in $\mathbf{A}^N_K$ with an isolated singularity at the origin. In the local analytic case we already know by Theorem 4(1) of [**11**] that for reduced hypersurfaces with an isolated singularity at the origin only $\Omega^{N-1}_{A/K}$ and $\Omega^{N}_{A/K}$ have nonzero torsion, where $N-1$ is the dimension of our hypersurface. The proof extends to the algebraic case as well. Since $\Omega^{N}_{A/K}$ is clearly cyclic on generator $\omega_1\wedge\cdots\wedge\omega_N$, it remains to consider the torsion submodule of $\Omega^{N-1}_{A/K}$.

The main result of this paper is an elementary proof that the torsion submodule of $\Omega^{N-1}_{A/K}$, henceforth denoted by $T(\Omega^{N-1}_{A/K})$, is a cyclic A-module and an explicit formula for its generator. We show

Theorem 1. *If A is a reduced affine hypersurface with an isolated singularity at the origin defined by a quasi-homogeneous polynomial F with weights λ_i and of (total) degree n, then $T(\Omega^{N-1}_{A/K})$ is a cyclic A-module generated by*

$$w_0=\sum(-1)^{i+1}(\lambda_i/n)x_i\,dx_1\wedge\cdots\wedge\hat{d}x_i\wedge\cdots\wedge dx_N.$$

We then proceed to show

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