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INTERPOLATION THEOREM FOR UNBOUNDED OPERATORS

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ABSTRACT. We prove the following unbounded generalization of the strong interpolation theorem [**2**, Corollary 3.16] under some extra hypotheses:

1. If h and k are self-adjoint operators on a Hilbert space H, k is bounded, $h \geq k$ and h_-, k_+ are compact, then there is a compact operator a such that $k \leq a \leq h$.

2. If h and k are self-adjoint operators on H, $h \geq k$ and h_-, k_+ are compact, then for all $\varepsilon > 0$ there is a compact operator a such that $k - \varepsilon 1 \leq a \leq h$.

Let H denote an infinite dimensional Hilbert space and $\mathcal{K}$ the space of compact operators on H. It is well known that there is one-to-one correspondence between bounded below self-adjoint operators, h, on H and densely defined closed quadratic forms, q_h, which are bounded from below given by $q_h(v) = (hv, v)$ on $D(q_h) = D([h - \lambda 1]^{1/2})$, $\lambda \leq h$. Given two densely defined quadratic forms q_1 and q_2, we write $q_1 \leq q_2$ if $D(q_1) \supset D(q_2)$ and $q_1(v) \leq q_2(v)$ for all $v \in D(q_2)$. This ordering defines an ordering of bounded below self-adjoint operators, i.e., we write $h \geq k$ if and only if $q_h \geq q_k$ where q_h and q_k are quadratic forms corresponding to h and k, respectively. As an extension of this ordering, if h is bounded below and k is bounded above self-adjoint operators on H, we will write $h \geq k$ if and only if $q_h(v) \geq q_k(v)$ for all v in $D(q_h) \cap D(q_k)$.

Now we introduce the notion of semi-continuous operators. Let A be a C^*-algebra and A^{**} its enveloping von Neumann algebra. For $M \subset A^{**}$, M_{sa} denotes the set of self-adjoint elements of M and M_{sa}^m the limits of increasing nets of elements of M_{sa}. We also denote by $\tilde{A}$ the C^*-algebra generated by A and the unit 1 of A^{**} and $Q(A)$ the quasi-state space of A, i.e., $Q(A) = \{\phi \in A^* \mid \phi \geq 0, ||\phi|| \leq 1\}$. Equipped with the weak* topology inherited from A^*, $Q(A)$ is a compact convex set. Recall that the evaluation map $\widehat{\ }$ on A_{sa}^{**} given by $\widehat{x}(\phi) = \phi(x)$ for x in A_{sa}^{**} and ϕ in $Q(A)$ is an order preserving isometry of A_{sa}^{**} onto the

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