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ON QUADRATIC SYSTEMS WITH A DEGENERATE CRITICAL POINT

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ABSTRACT. We study phase portraits of quadratic systems with exactly two critical points, one of them degenerate. This problem has already been considered in [**10**], where part of the results are obtained by computer. Here we deal with these systems in terms of semi-complete families of rotated vector fields. This new approach allows us to prove most of the bifurcation diagrams that we obtain.

1. Introduction. A quadratic system, QS, is a system of two real autonomous differential equations

$$\dot{x} = P(x,y), \qquad \dot{y} = Q(x,y), \tag{1}$$

where the dot indicates derivative in respect to a real variable t, and P and Q are polynomials in two variables with degree at most two and not both with degree less than or equal to one. We denote by $X = (P,Q)$ the vector field associated to (1).

QS gives the simplest example of nonlinear differential equations and also presents most of the difficulties that nonlinear systems have. For instance, it was not proved that a given QS had a finite number of limit cycles until 1987 (see [**4**]). Nothing is known about the maximum number of limit cycles that a QS may have, except that it is greater than or equal to four. Hence, both the simplicity and the complexity that QS present, have been the reasons for which particular subfamilies of such systems have been extensively studied. In this way we can recall, for instance, the following subfamilies: homogeneous QS, QS with a start nodal point, bounded QS, QS without finite critical points, QS with an invariant straight line, QS with a degenerate critical point, QS with exactly one critical point, QS with a weak focus, etc., (see [**14**]).

In the above subfamilies two fundamental questions are studied: the phase portraits of the QS and the number of limit cycles that they may have.

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