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A SURVEY ON PARACOMPLEX GEOMETRY

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1. Introduction. We shall call paracomplex geometry the geometry related to the algebra of paracomplex numbers [**70**] and, mainly, the study of the structures on differentiable manifolds called paracomplex structures. When, moreover, we consider a compatible neutral pseudo-Riemannian metric, we have the para-Hermitian and para-Kähler structures, and their variants.

This subject has been studied, since the first papers by Rashevskij [**94**], Libermann [**69**] and Patterson [**90**] until now, from several different points of view. Moreover, the papers related to it have appeared many times in a rather disperse way, and the different schools or authors have worked many times having no relation with one another. So, the interest for a survey on the topic is clear; and, of course, mathematicians, and in particular differential geometers, will find a lot of definitions, examples, results and references in paracomplex geometry. Furthermore, we think that the solved problems in paracomplex geometry are only a little part of the "first" questions to be studied in the theory. And this is one of the attractive features of it: to be *de facto* a young branch of differential geometry, with many significant results to be proved, as the reading of the present survey, we hope, shows. Nowadays, the subject has already applications to several topics, as: negatively curved manifolds and Anosov diffeomorphisms [**47**], quantizable coadjoint orbits [**57**], mechanics [**11, 86**], chronogeometry [**82**], elliptic geometry [**32**], pseudo-Riemannian space forms [**38**], etc.

On the other hand, as is seen in certain recent works, paracomplex geometry is related to some physical problems, and we think that this work can also be useful for physicists and mathematicians working in them. In particular, since every almost para-Kählerian manifold is symplectic, those manifolds furnish a large family of symplectic manifolds.

Moreover, as Kaneyuki [**49**] points out, the study of global geometric

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