ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 1, Winter 1996

# COVERING CONGRUENCES IN HIGHER DIMENSIONS

TODD COCHRANE AND GERRY MYERSON

ABSTRACT. We construct a set of covering congruences for the set of all ordered pairs of integers.

Erdős popularized the notion of a *set of covering congruences* (hereinafter, a *cover*). This is a finite set $(a_1, m_1), \dots, (a_r, m_r)$ of ordered pairs of integers with $1 < m_1 < \cdots < m_r$ such that every integer $x$ satisfies at least one of the congruences $x \equiv a_j \pmod{m_j}$. The simplest example is $(0,2), (0,3), (1,4), (1,6), (11,12)$.

It is obvious that there does not exist a *homogeneous* cover, that is, one in which $a_j = 0$ for all $j$ (what homogeneous congruence is satisfied by 1?). Our purpose is to show that there is a homogeneous cover for the group of all ordered pairs of integers, that is,

**Theorem.** *There is a finite set of ordered triples* $(a_1, b_1, m_1), \dots, (a_r, b_r, m_r)$ *with* $1 < m_1 < \cdots < m_r$ *and with* $GCD(a_j, b_j, m_j) = 1$ *for all* $j$ *such that every pair of integers* $(x, y)$ *satisfies at least one of the congruences* $a_j x - b_j y \equiv 0 \pmod{m_j}$.

The GCD condition is needed to weed out covers such as (1,0,2), (2,2,4), (0,3,6), in which repeated moduli are disguised by common factors. The theorem may not be too surprising, in view of the obvious correspondence between the one-variable congruence $x \equiv a \pmod{m}$ and the two-variable homogeneous congruence $x - ay \equiv 0 \pmod{m}$. But this correspondence, applied directly to a cover of the integers, does not produce a homogeneous cover of ordered pairs (at any rate, we don't see how it does), so a further idea is necessary. Such an idea is contained in Lemma 1, below.

Covering congruences in higher dimensions are discussed by Schinzel [**6**] and Fabrykowski [**2**]. Porubský [**5**] published a thorough survey of

Received by the editors on April 8, 1994, and in revised form on September 5, 1994.
1991 *Mathematics Subject Classification.* 11A07, 11B25.
*Key words and phrases.* Covering congruences.