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THE ASYMPTOTIC BEHAVIOR OF A REDUCIBLE SYSTEM OF NONLINEAR INTEGRAL EQUATIONS

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ABSTRACT. The methods developed in this paper are motivated mainly by the study of models for rabies. Rabies is a multispecies disease in which the virulence of the virus, and its affect on different species, leads to models where the infection matrix is reducible.

The asymptotic behavior of a reducible system of nonlinear integral equations describing the spatio-temporal development of such an epidemic is studied. When the system is nonreducible, an approximate saddle point method can be used for a restricted model with constant infection and removal rates. This approximate method [**17**] indicated that the asymptotic speed of propagation is c_0, the minimum wave speed. A rigorous analytic proof of this result was given subsequently in Radcliffe and Rass [**18**].

A reducible set of types may be considered as split into nonreducible subsets of types, so that within each subset all types may infect every other type, possibly through a series of infections. For any two subsets, infection in at least one subset cannot cause infection to occur in the other subset. Consider an infection in the ith subset only, the density of types in the other subsets being taken to be zero. Let c_i be the corresponding asymptotic speed of propagation. Then, for the full system, the asymptotic speed of propagation differs for the different subsets. Each subset infected will force the epidemic in any subset it infects to propagate with at least its speed of propagation. The approximate saddle point method was again used for the restricted reducible model [**20**]. It indicated that, for a particular subset, the asymptotic speed of propagation is the maximum of the c_i over all subsets i which can cause an infection in the particular subset, and which can themselves be infected by the initial infection in the system.

In this paper, with certain conditions imposed, a rigorous proof of these results is obtained for the general reducible model. It is remarkable that these conditions cover not only all cases in which the saddle point method can be applied but

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