ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 2, Spring 1996

ULTRAFILTERS OVER N AND OPERATORS ON L^1

MINOS A. PETRAKIS

Introduction. Let X be a Banach space. B_X denotes the unit ball of X and X^* denotes the dual of X. Let (Ω, Σ, μ) be a probability space. We denote by $L^1(\mu)$ the Banach space of all μ-integrable functions with the usual norm. L^1 denotes the space of Lebesgue integrable functions on the unit interval $[0,1]$. For a nonnegligible subset A of Ω, $P(A)$ is the set $\{f \in L^1(\mu) : f \geq 0, \operatorname{supp}(f) \subseteq A \text{ and } \int f\, d\mu = 1\}$ of all probability densities supported in A.

A *tree* in X is a bounded family $(x_{n,k})$, $n = 0, 1, \dots,$ $k = 1, 2, \dots 2^n$ of elements of X verifying $x_{n,k} = (x_{n+1,2k-1} + x_{n+1,2k})/2$ for each $n = 0, 1, 2, \dots,$ $k = 1, 2, 3, \dots, 2^n$.

A δ-*tree* is a tree verifying $||x_{n+1,2k-1} - x_{n+1,2k}|| \geq \delta$ for each $n = 0, 1, 2, \dots,$ $k = 1, 2, \dots, 2^n$.

A δ-*Rademacher tree* is a tree $(x_{n,k})$ verifying $||\sum_{k=1}^{2^n}(-1)^{k+1}x_{n,k}|| \geq \delta 2^n$.

Let $I_{n,k} = [(k-1)/2^n, k/2^n]$, $n = 0, 1, 2, \dots, k = 1, 2, \dots, 2^n$ and $h_{n,k} = 2^n \cdot \mathbf{X}_{n,k}$ where $\mathbf{X}_{n,k}$ is the characteristic function of the dyadic interval $I_{n,k}$. If $T : L^1 \to X$ is a (bounded) operator it is clear that $(T(h_{n,k}))$, $n = 0, 1, 2, \dots,$ $k = 1, 2, \dots, 2^n$ is a tree in X. Conversely, every tree $(x_{n,k})$ in X produces an operator $T : L^1 \to X$ such that $T(h_{n,k}) = x_{n,k}$, $n = 0, 1, \dots,$ $k = 1, 2, \dots 2^n$.

An operator $T : L^1(\mu) \to X$ is called *Dunford-Pettis* if it maps weakly convergent sequences in $L^1(\mu)$ into norm convergent sequences in X.

A Banach space X has the *complete continuity property* if every operator from L^1 into X is Dunford-Pettis.

In [**2**] it is proved that if $T : L^1 \to X$ is an operator and $||T(r_n)|| > 2\varepsilon$ for some L^∞-bounded sequence (r_n) and some $\varepsilon > 0$, then there exists a set A of positive Lebesgue measure such that $\limsup_n ||T(r_n \cdot f)|| \geq \varepsilon$ for all f in $P(A)$. This result was used in [**2**] and [**4**] in the construction

Received by the editors on April 1, 1994.
AMS *Subject Classification.* 46B20, 47B38, 46B22, 47B07.

Copyright ©1996 Rocky Mountain Mathematics Consortium