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THE SPECTRAL THEORY OF SECOND ORDER TWO-POINT DIFFERENTIAL OPERATORS III. THE EIGENVALUES AND THEIR ASYMPTOTIC FORMULAS

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ABSTRACT. In the third part of a four-part series, the eigenvalues of a two-point differential operator L in $L^2[0,1]$ are calculated, along with the corresponding asymptotic formulas. L is determined by a formal differential operator $l = -D^2 + q$ and by independent boundary values B_1, B_2. The rates of convergence in the asymptotic formulas vary with the form of B_1, B_2 (Cases 1–4) and with the smoothness of q.

1. Introduction. In this paper, which is the third part in a four-part series, we continue our development of the spectral theory for a linear second order two-point differential operator L in the complex Hilbert space $L^2[0,1]$. In Part I [**14**] a priori estimates for the eigenvalues of L are derived, and the generalized eigenfunctions are shown to be complete. In Part II [**15**] the characteristic determinant of L is constructed utilizing operator theory methods. Using this representation of the characteristic determinant, here in Part III we calculate the actual eigenvalues of L, compute the corresponding algebraic multiplicities and ascents, and determine asymptotic formulas for the eigenvalues. We also establish the geometries and the growth rates for the characteristic determinant, which are the key results needed for Part IV where the L^2-expansion theory is developed.

Let L be the differential operator in $L^2[0,1]$ defined by

$$\mathcal{D}(L) = \{u \in H^2[0,1] \mid B_i(u) = 0, i = 1,2\}, \qquad Lu = lu,$$

where

$$l = -\left(\frac{d}{dt}\right)^2 + q(t)\left(\frac{d}{dt}\right)^0$$

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