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THE PENNEY-FUJIWARA PLANCHEREL FORMULA FOR GELFAND PAIRS

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ABSTRACT. This paper is concerned with the Penney-Fujiwara Plancherel formula for the quasi-regular representation of a homogeneous space G/H. The specific spaces considered are semidirect products $G = H \ltimes N$, where H is compact and N is simply connected nilpotent. An explicit description is given for the Plancherel formula, and the attendant Penney distributions associated with it, in the important special case that (G, H) is a *Gelfand pair*. The description is in orbital parameters, and thereby supplies an important instance of the validity of the Kirillov orbit method that is outside the usual context of exponential solvable groups. Other results that are found in the paper include: (i) a proof of the fact that a distribution-theoretic form of Frobenius reciprocity holds for Gelfand pairs; (ii) the demonstration that the well-known form of the Penney distributions in terms of *real* polarizations continues to be valid when the representations are realized by *complex* polarizations; (iii) an explicit formula for the intertwining operator for the direct integral decomposition of the quasi-regular representation of G/H; and (iv) an orbital criterion for the quasi-regular representation to be multiplicity-free, as well as a criterion for an irreducible to occur in the spectrum.

0. Introduction. This paper is devoted to the Plancherel formula which describes the direct integral decomposition of a quasi-regular representation. The latter is the natural representation of a Lie group G on the square-integrable functions on a homogeneous space of G. More precisely, if H is a closed subgroup of G, the corresponding quasi-regular representation is nothing more than the representation τ of G obtained by inducing the identity representation of H up to G. The *abstract* or "soft" Plancherel formula is a statement of unitary equivalence between τ and a direct integral of irreducible unitary representations

$$\tau \cong \int_{\mathcal{X}}^{\oplus} n(\pi)\, \pi \, d\nu_H(\pi). \tag{0.1}$$

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