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CAUCHY TRANSFORMS OF MEASURES AND WEIGHTED SHIFT OPERATORS ON THE DISC ALGEBRA

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ABSTRACT. We consider families $\mathcal{F}_\alpha$, $\alpha > 0$, of analytic functions $F_\mu(z)$ on the unit disc that are obtained by integrating $(1 - e^{i\theta}z)^{-\alpha}$ with respect to complex measures μ on the unit circle. These families are Banach spaces which are isometrically isomorphic to the dual space of the disc algebra. The collection $\mathfrak{M}_\alpha$ of all multipliers of $\mathcal{F}_\alpha$ is shown to be the set of adjoints of the commutant of a certain weighted shift operator on the disc algebra.

It is known that if $0 < \alpha < \beta$, then $\mathfrak{M}_\alpha \subset \mathfrak{M}_\beta$. We show that this inclusion is proper in a number of cases. Also, for various α we find conditions on the sequence of Taylor coefficients of an analytic function that imply that the function is a multiplier of $\mathcal{F}_\alpha$.

1. Introduction. In this paper we consider families $\mathcal{F}_\alpha$ of Cauchy transforms of complex Borel measures on the unit circle $\mathbf{T}$ in the complex plane. For $\alpha > 0$ let $\mathcal{F}_\alpha$ consist of all functions f on the unit disc of the form

$$F_\mu(z) = \int_{\mathbf{T}} \frac{1}{(1 - e^{i\theta}z)^\alpha}\, d\mu(e^{i\theta}),$$

where μ is a complex Borel measure on $\mathbf{T}$. If $[\mu]$ is the equivalence class of all measures representing F_μ, then the norm $||F_\mu||_\alpha = ||[\mu]||$ makes $\mathcal{F}_\alpha$ into a Banach space.

The family $\mathcal{F}_1$ is of classical interest [**8, 9, 20**]. For example, it includes the Hardy spaces H^p for $p \geq 1$. The families $\mathcal{F}_\alpha$, $\alpha > 0$, were defined by T.H. MacGregor [**16**] in connection with geometric function theory. In particular, [**16**] includes the result that for $0 < \alpha < \beta$, $\mathcal{F}_\alpha \subset \mathcal{F}_\beta$. We give a proof of the stronger result that $\mathcal{F}_\alpha \subset \mathcal{F}_{\beta a}$, the subset of $\mathcal{F}_\beta$ consisting of transforms of absolutely continuous measures,

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