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THE BAER-KAPLANSKY THEOREM FOR A CLASS OF GLOBAL MIXED GROUPS

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1. Introduction. This paper considers the case of isomorphic endomorphism rings $E(G)$ and $E(G')$ of global, mixed abelian groups G and G'. Throughout, all groups are understood to be abelian, and all isomorphisms $E(G) \cong E(G')$ ring isomorphisms. Before describing new results, we recount some of the history of our general problem. According to the Baer-Kaplansky theorem [**4**, Theorem 28], if G and G' are torsion groups, then every isomorphism $E(G) \cong E(G')$ is induced by an isomorphism of the groups themselves. Kaplansky's proof of this result plays off the abundance of cyclic direct summands of reduced p-groups: primitive idempotents in $E(G)$ correspond to direct summands of G' under the ring isomorphism, and an isomorphism $G \to G'$ can be constructed by utilizing a carefully chosen set of such idempotents. By observing that Kaplansky's method also works in the torsion-free case if each group possesses a cyclic, nonzero direct summand, Wolfson [**16**] subsequently proved a similar theorem for torsion-free modules over the p-adic integers. Not surprisingly, the proofs of such theorems in the mixed case require quite different techniques, because idempotents in the endomorphism rings no longer suffice to recover the full structure of the underlying groups and modules. Suitable methods were developed in a sequence of papers by May [**9**, **5**–**8**], and a wide variety of Baer-Kaplansky type theorems were put forth by him for local, mixed groups and modules in these accounts. This author's framework now seems indispensable for dealing with the problem of isomorphic endomorphism rings of many classes of mixed groups, even in the little-explored global case. The method we will use of embedding both groups in the completion of a single torsion group (Lemma 1) is due to him.

The class $\mathcal{G}$ of global mixed groups that we shall consider is described below. We show that many nonisomorphic groups $G' \in \mathcal{G}$ with $E(G') \cong E(G)$ are possible for groups G in the class. Nevertheless,

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