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ON CONVERGENCE OF CONDITIONAL EXPECTATION OPERATORS

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ABSTRACT. Given an operator $T : U_X(\Sigma) \to Y$ or $T : C(H, X) \to Y$, one may consider the net of conditional expectation operators (T_π) directed by refinement of the partitions π. It has been shown previously that (T_π) does not always converge to T. This paper gives several conditions under which this convergence does occur, including complete characterizations when $X = \mathbf{R}$ or when X^* has the Radon-Nikodým property.

1. Introduction. It is well known that if $T : U_X(\Sigma) \to Y$ is a bounded linear operator, where $U_X(\Sigma)$ is the uniform closure of the X-valued Σ-simple functions, then there is a unique finitely additive set function $m : \Sigma \to L(X, Y)$ with finite semi-variation such that $T(f) = \int f\, dm$ for all $f \in U_X(\Sigma)$. Also, if $T : C(H, X) \to Y$, there is a unique weakly regular $m : \beta(H) \to L(X, Y^{**})$ such that $T(f) = \int f\, dm$ for $f \in C(H, X)$. In each case $\tilde{m}(H) = ||T||$. Given such an operator T, a finite partition π of H, and a measure μ on H, a conditional expectation operator T_π can be defined. It was shown in [**1**] that the net (T_π) directed by refinement does not always converge to T in the operator norm. Conditions under which this convergence does occur are discussed herein.

Throughout, X and Y are Banach spaces. The closed unit ball of X is denoted by B_X. We will use H for a compact Hausdorff space and $C(H, X)$ for the space of continuous functions from H to X. An arbitrary σ-algebra of subsets of some universal space Ω will be represented by Σ, and when $\Omega = H$, we will use $\Sigma = \beta(H)$, the Borel sets of H, without further mention. An additive set function $m : \Sigma \to X$ will be called a vector measure, while by a measure we mean a countably additive set function $\mu : \Sigma \to [0, \infty)$.

For a vector measure $m : \Sigma \to X$, we define the variation of m as usual and the scalar semi-variation $||m||$ of m as in [**8**]. If $m : \Sigma \to L(X, Y)$,

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