ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 2, Spring 1996

PIECEWISE MONOTONIC DOUBLING MEASURES

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1. Introduction. A positive Borel measure μ, defined on $\mathbf{R}$ or on some interval, is a doubling measure if there exists a constant C such that for each interval I, $\mu(2I) \leq C\mu(I)$, where $2I$ is the interval with the same center as I and twice the length. Somewhat surprisingly, doubling measures are not necessarily absolutely continuous–Beurling and Ahlfors [**2**] constructed a singular doubling measure.

If a doubling measure is absolutely continuous, its Radon-Nikodym derivative is called a doubling weight. An important class of doubling weights is (A_∞). For $p > 1$ a nonnegative function w is an (A_p) weight if

$$\sup_I \left(\frac{1}{|I|}\int_I w\,dx\right)\left(\frac{1}{|I|}\int_I w^{1-p'}\,dx\right)^{p-1} < \infty,$$

where the supremum is taken over all intervals I and p' is the conjugate exponent of p. If $Mw(t) \leq Cw(t)$ almost everywhere, where Mw is the Hardy-Littlewood maximal function of w, then w is an (A_1) weight. The union of the (A_p) classes is denoted by (A_∞). Not every doubling weight is an (A_∞) weight; C. Fefferman and Muckenhoupt [**4**] and more recently Wik [**14**] have given counter-examples.

In this paper we study those doubling measures which are piecewise monotonic. A measure μ is *monotonic* if the measure of a right translate of a set is always larger (or smaller) than the measure of the set itself, and μ is *piecewise monotonic* if its support is the union of a (finite) number of intervals on which μ is monotonic. We show that piecewise monotonic doubling measures are absolutely continuous and their Radon-Nikodym derivatives are (A_∞) weights.

The paper is organized as follows. In Section 2 we determine the singular parts of monotonic measures and show that piecewise mono-

Received by the editors on September 6, 1993, and in revised form on May 19, 1994.
The majority of the results in this paper are from a Ph.D. dissertation written under the direction of Donald Sarason at the University of California at Berkeley. I want to thank him for his patience and support. Early parts of this work were supported under a National Science Foundation Graduate Fellowship.