ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 2, Spring 1996

L^p MATRIX COEFFICIENTS FOR NILPOTENT LIE GROUPS

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ABSTRACT. We show that if G is a connected, simply connected nilpotent Lie group, then there is a fixed number p such that if π is any irreducible unitary representation of G, then some (equivalently, a dense set of) matrix coefficients are L^p functions on G mod the kernel of π.

1. Introduction. In this paper we undertake a study of one aspect of the asymptotic behavior of the matrix coefficients of irreducible unitary representations of Lie groups. The behavior at infinity of these matrix coefficients often gives important information about the structure of the irreducible representations themselves and about harmonic analysis in general. For example, detailed asymptotic estimates on the behavior of matrix coefficients of irreducible representations of semi-simple groups (real and p-adic) have played a central role in the work of Harish-Chandra, Langlands, and others. (Examples of fairly recent results for these groups are given in [**1, 3**] and [**11**].) A related example where the asymptotics of matrix coefficients plays a key role is the Kunze-Stein L^p convolution theorem [**9**]. Again, square integrable representations, or discrete series representations, which are characterized by the asymptotic behavior of their matrix coefficients (namely square integrability, see below) are a very important class of representations, since they are exactly the irreducible representations that appear as summands in the regular representation. (For a proof of this fact, see Section 14 of [**5**].) These representations are also the fundamental building blocks for all unitary representations of semi-simple groups, and they can play a similar role in other Lie groups as well; see [**10**].

For more general groups, it was proved in [**8**] that the matrix coefficients of any irreducible unitary representation π of a real algebraic

Received by the editors on November 25, 1991, and in revised form on December 7, 1994.
This joint research by the two authors was completed prior to Larry Corwin's tragic and untimely death in March of 1992, and this publication is his last research paper. He is deeply missed by his many friends, colleagues and coworkers. CCM
Supported by NSF grants DMS91-05789 and DMS89-12907 respectively.