ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 2, Spring 1996

CLOSED GEODESICS ON IDEAL POLYHEDRA OF DIMENSION 2

C. CHARITOS

1. Introduction. In this paper we are concerned with ideal polyhedra of dimension 2. These spaces consist of ideal hyperbolic triangles which are glued together by isometries along their sides. It is important to define and begin the study of ideal polyhedra in the context of negatively curved polyhedra introduced by Gromov in [**7**]. Ideal polyhedra of dimension 2 appear naturally as the 2-skeleton of 3-manifolds obtained by gluing together ideal tetrahedra. Topologically, simplicial complexes with vertices removed are examples of spaces which are homeomorphic to ideal polyhedra.

There is a consistent way to define a length pseudo-distance d_K on an ideal polyhedron K. Theorem 1 gives necessary conditions for this length pseudo-distance to be a geodesic metric. This result is analogous to a theorem of Bridson for geometric complexes (see [**1**, Theorem 1.1] or [**9**, Theorem 3.6]).

By introducing the concept of the developing surface along a curve γ in K we prove the existence of a closed geodesic in the free homotopy class of a closed curve which is not homotopic to a point or to a cusp in K. We prove this by elementary methods; we don't use any shortening process [**4**, Chapter 10, **5**] but we reduce the proof to the case of surfaces.

In Propositions 2 and 3 we prove simple properties of the universal covering $\tilde{K}$ of K. In particular, in Proposition 3 we prove that every local geodesic of $\tilde{K}$ is a geodesic; this is not true in general for two-dimensional simply connected polyhedra of negative curvature.

Finally, Propositions 2 and 3 permit us to establish the uniqueness of a closed geodesic in its free homotopy class. This is Theorem 2 and is the main application of the method outlined above.

2. Ideal polyhedra of dimension 2. We begin with the precise

Received by the editors on March 16, 1994.

Copyright ©1996 Rocky Mountain Mathematics Consortium