ON MARTINDALE'S THEOREM

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ABSTRACT. Martindale's theorem characterizes prime rings which satisfy a generalized polynomial identity. In the present paper we give further characterizations in terms of the ring only, eliminating thereby the central closure from the formulation of Martindale's theorem.

The definition of a generalized polynomial identity (GPI) and Martindale's characterization of a prime ring satisfying a GPI both make use of the central closure of the ring. Here we give internal characterizations, eliminating thereby the central closure. The possibility of such a characterization may belong to the 'folklore' for specialists in the field; we could not find it in the literature, however, and we think it can be of interest for the nonspecialist because it yields a conceptually simpler approach.

Notice also that the extended centroid, and through it the usual definition of a GPI, is connected with the maximal ring of left quotients of the ring and hence has a slight one-sided flavor, whereas conditions 4 and 5 of our theorem are obviously two-sided.

Theorem. *For a prime ring R, the following conditions are equivalent.*

(1) *R satisfies a GPI,*

(2) *R has a square-cancellable element a such that Ra is a uniform left ideal and aRa is a domain of finite rank over its center,*

(3) *R has a left ideal $_RL$ such that $\operatorname{End}_R L$ is a domain of finite rank over its center,*

(4) *R has an element a such that $a^2 \neq 0$ and aRa is a PI-ring,*