ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 2, Spring 1996

FACTORIZATION IN COMMUTATIVE RINGS WITH ZERO DIVISORS

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ABSTRACT. The purpose of this paper is to study factorization in commutative rings with zero divisors with particular emphasis on how the theory of factorization in integral domains is similar to and different from the theory for commutative rings. Various notions of "associate" are considered. Each form of "associate" leads to a type of "irreducible" element, and each type of "irreducible" element leads to a form of atomicity (elements being products of that type of irreducible element) and unique factorization. Numerous examples are given, including an example of an atomic LCM ring which does not satisfy ACCP or have unique factorization. Factorization in polynomial rings and power series rings is considered.

1. Introduction. A fundamental theme in algebra is the factorization of elements into irreducible elements. The setting is usually a commutative integral domain R with identity. A nonzero, nonunit element a of R is said to be irreducible if for any factorization $a = bc$, either b or c is a unit. There are then two natural questions: (1) What integral domains have the property that every nonzero, nonunit element has a factorization into irreducible elements? and (2) What uniqueness properties, if any, do factorizations into irreducible elements have? As to the first question, usually some chain condition, such as the ascending chain condition on principal ideals, is used to show that every element has a factorization into irreducible elements. The second question is more complicated. Factorizations can be highly nonunique. The class group of a Dedekind domain (or more generally, the divisor class group of a Krull domain) in some sense measures the deviation from having unique factorization. For a discussion of the possible lengths of factorizations, see Anderson and Anderson [**5**].

Much of the theory of factorization in an integral domain can be generalized to commutative rings with zero divisors, often in several ways. Some of this has already been done in a series of papers by

Received by the editors on November 22, 1993.
1991 *Mathematics Subject Classification.* 13A05, 13A15, 13F99.

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