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DECAYING SOLUTIONS OF ELLIPTIC SYSTEMS IN R^n

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ABSTRACT. We consider nonlinear elliptic systems, with prototype form: $-\Delta\vec{u} = \lambda\vec{f}(x,\vec{u})$ in $\mathbf{R}^n$ and show the existence of positive decaying (componentwise) solutions. Our basic tools are classical estimates of Gidas, Ni, Nirenberg and Egnell coupled with Leray-Schauder degree theory arguments in weighted spaces. We do not assume, in general, that the system is variational, although mountain pass arguments are employed for one such case. This approach enables us to obtain, in particular, the existence of positive solutions also for reducible systems, and the extension of several recent results, some even in the scalar case.

1. Introduction. This paper deals with elliptic nonlinear systems formally given by the equation

$$(1)\qquad -\Delta\vec{u} = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_M \end{pmatrix} \vec{f}(x,\vec{u})$$

in $\mathbf{R}^n$, $n \geq 3$, and the related problem $-\Delta\vec{u} = \lambda\vec{f}$, i.e., $\lambda_1 = \cdots = \lambda_M = \lambda$. Here $\vec{u} = (u_1,\dots,u_M)$, and we are interested in the evidence of positive (componentwise) solutions $\vec{u}$ to (1) such that $\vec{u} \to \vec{0}$ at ∞, in the case that $\vec{f}(x,\vec{u})$ is superlinear and subcritical. We do not usually require that (1) admit a variational structure, although some results are obtained under this assumption.

The rough outline of this paper is as follows: We first give conditions under which (1) has solutions for all $\vec{\lambda} = (\lambda_1,\dots,\lambda_M) > \vec{0}$. Our approach here is based on the observation that, under suitable conditions, the fundamental scalar results of Gidas, Ni and Nirenberg [**12**], Gidas and Spruck [**13**] and Egnell [**10**] can be employed to show that all positive solutions of (1) have a norm which is bounded and bounded

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