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A NEW SUFFICIENT CONDITION FOR THE DENSENESS OF NORM ATTAINING OPERATORS

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ABSTRACT. We give a new sufficient condition for a Banach space Y to satisfy Lindenstrauss's property B, namely the set of norm-attaining operators from any other Banach space X into Y is dense. Even in the finite-dimensional case, our result gives new examples of Banach spaces with property B.

Introduction. As a special case of the Bishop-Phelps theorem [**5, 6**], the set of norm-attaining functionals on a Banach space is dense in the dual space for the norm topology. In their earlier paper [**5**], Bishop and Phelps addressed the question of what Banach spaces might play the role of the scalar field in their theorem. Intensive research on this question was initiated by J. Lindenstrauss [**15**], who introduced the so-called property B. Given two Banach spaces X and Y, let us consider the Banach space $L(X,Y)$ of bounded linear operators from X into Y, and let us denote by $NA(X,Y)$ the set of norm-attaining operators. Thus, $T \in NA(X,Y)$ means that there is some $x \in S_X$ (the unit sphere of X) such that $||Tx|| = ||T||$. The Banach space Y is said to satisfy property B if $NA(X,Y)$ is dense in $L(X,Y)$ for all Banach spaces X. Therefore, the Bishop-Phelps theorem gives that the scalar field has property B. We are not concerned in this paper with the corresponding property A (when Y is taken as the domain space) also introduced by Lindenstrauss in [**15**]. The interested reader may consult the papers by J. Bourgain [**7**], C. Stegall [**19**] and W. Schachermayer [**18**]. Unlike property A, present knowledge of property B is far from being satisfactory. Let us give a brief outline of known results and some open problems.

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