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RINGS OF ENDOMORPHISMS OF SEMIGROUP-GRADED MODULES

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ABSTRACT. We analyze various types of endomorphism rings of graded modules over rings graded by finite semigroups, and show connections between such rings and certain skew rings. In the specific case of group-graded rings our results will yield the isomorphism theorems of Albu and Năstăsescu.

0. Introduction. In this article we continue the investigation of *rings graded by semigroups* which was begun in [**1**] and [**3**]. Specifically, we investigate various rings of endomorphisms associated with certain types of graded modules. We briefly sketch our course of study in the next few paragraphs.

For a ring R graded by the semigroup S, and graded left R-module ${}_RN$, we define the graded module $U(N)$ to be a direct sum of graded modules of the form $N(f)$ (where $f \in S^* = S - \{z\}$). Our interest in such modules stems from their importance within the category $R - gr$ of S-graded left R-modules; for instance, as shown in [**3**, Corollary 3.2], modules of the form $U(R)$ are often progenerators for $R - gr$. We will analyze the three rings

$$\mathrm{End}_{R-gr}(U(N)), \quad \mathrm{END}_R(U(N)) \quad \text{and} \quad \mathrm{END}_R^{-1}(U(N)).$$

These are, respectively, the ring of graded endomorphisms of $U(N)$, the direct sum of the groups of endomorphisms of $U(N)$ of degree f (where f ranges in S^*), and the direct sum of the groups of endomorphisms of $U(N)$ of degree f^{-1} (where f again ranges in S^*). The

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