ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 3, Summer 1996

AN EFFECTIVE ROTH'S THEOREM FOR FUNCTION FIELDS

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ABSTRACT. We will give a new proof of Roth's theorem for function fields which is motivated by Steinmetz's proof of Nevanlinna's second main theorem of slowly moving target functions. This method provides effective results.

0. Introduction. The correspondence between number theory and value distribution theory has been observed by Osgood [**3**] and Vojta [**6**]. Both these areas are related to function fields. For example, one can establish the analogue of Cartan's truncated second main theorem for function fields [**7**], which corresponds to the so-called abc conjecture for number fields. Usually for a corresponding result in function fields, one can also expect two proofs, one analogous to number theory and the other analogous to value distribution theory. For example, the Thue-Siegel-Roth theorem for function fields was proved by Uchiyama [**5**] with a line of proof similar to the one for number fields, and hence is ineffective. However, Roth's theorem for function fields should also correspond to some sort of second main theorem with moving target functions in value distribution theory (such as in Nevanlinna's conjecture with slowly moving target functions) which was proved by Steinmetz [**4**] in the case of functions. Indeed, since the ideas in [**4**] mainly involve Wronskians, one can expect an analogous proof for function fields. In this paper we will give a proof of Roth's theorem for function fields which is analogous to [**4**]. As we move on to the proof, it will become clear that the results are effective in the sense that the constants in the proof can be effectively determined from the method of the proof.

Let K be the function field of a smooth projective curve C over an algebraically closed field k of characteristic 0. For each $P \in C$ we have a valuation v_P on K. Each v_P can be extended to K^a, where K^a is the algebraic closure of K. For each $f \in K$ one can define the height $h_K(f) = \sum_{P\in C} -\min\{0, v_P(f)\} = \sum_{P\in C} \max\{0, v_P(f)\}$. We

Received by the editors on August 17, 1994, and in revised form on May 1, 1995.