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NONSINGULAR ZEROS OF QUINTIC FORMS OVER FINITE FIELDS

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ABSTRACT. Let f be a nondegenerate homogeneous polynomial of degree 5 defined over the finite field $\mathbf{F}_q$ in at least six variables. We show that if $q > 101$, then f has a nonsingular $\mathbf{F}_q$-rational zero.

1. Introduction. Let $f \in \mathbf{F}_q[x_1, \dots, x_n]$ be a homogeneous form of degree d, where $\mathbf{F}_q$ is the finite field with q elements. If $n > d$, then one knows (see Lemma 2.2) that f has a nontrivial $\mathbf{F}_q$-rational zero. It is also useful to know under what conditions one can guarantee the existence of a *nonsingular* $\mathbf{F}_q$-rational zero of f.

To cite one example, suppose F is a polynomial defined over a p-adic field K with coefficients in the ring A of integers of K, and let $\overline{F}$ denote the reduction of F modulo the maximal ideal of A. Then $\overline{F}$ is defined over some finite field. If $\overline{F}$ has a nonsingular zero over this finite field, then Hensel's lemma allows one to construct a nonsingular K-rational zero of F.

A theorem of Lang-Weil [**4**, p. 824, Corollary 3] says that if f is absolutely irreducible and q is sufficiently large (depending on n and d), then f has a nonsingular zero over $\mathbf{F}_q$. But other than for the well-known and easy cases of a quadratic or cubic form (see the end of Lemma 3.5 for the cubic case), the only other effective general result comes from Deligne's solution of the Weil conjectures, from which one can obtain a bound on q for nonsingular f (see [**2**, p. 276, Theorem 6.1]).

Our main result is the following (Corollary 4.5):

Let f be a nondegenerate quintic form in at least six variables over the finite field $\mathbf{F}_q$. If $q > 101$, then f has a nonsingular $\mathbf{F}_q$-rational zero.

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