ON MAHLER’S CLASSIFICATION
IN LAURENT SERIES FIELDS

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ABSTRACT. In 1932, K. Mahler introduced his famous classification for complex numbers in disjoint sets \( A, S, T, U \) [9, 10]. In 1978, P. Bundschuh introduced a similar classification for the field of formal Laurent series over a finite field \( K \) and gave some explicit series in the class \( U \). Here we consider the case of an arbitrary field \( K \) and prove the existence of \( U \)-numbers whose continued fractions verify additional properties.

0. Introduction. T. Schneider’s book [16, Chapter 3] is a complete introduction to the subject, whereas A. Baker’s book [2, Chapter 8] offers a general outlook. For a polynomial \( P = c_n x^n + \cdots + c_0 \) in \( \mathbb{Z}[X] \) with \( c_n \neq 0 \), we define the degree \( d(P) \) and the height \( h(P) \) by

\[
d(P) = n, \quad h(P) = \text{Max} \{ |c_j|, 0 \leq j \leq n \}.
\]

For natural numbers \( n \geq 1, H \geq 1 \), we consider

\[(1) \quad P_{n,H} = \{ P \in \mathbb{Z}[X] : d(P) \leq n, H(P) \leq H \}\]

and for any complex number \( \xi \), we define \( w(n, H, \xi), w_n(\xi), w(\xi) \) by

\[
\text{Min} \{ |P(\xi)| : P \in P_{n,H} \} = H^{-n} w(n, H, \xi)
\]

and

\[
w_n = w_n(\xi) = \limsup_{H \to \infty} w(n, H, \xi);
\]

\[
w = w(\xi) = \limsup_{n \to \infty} w_n(\xi)
\]

\[(2) \quad v = v(\xi) = \text{Inf} \{ n : w_n(\xi) = \infty \}
\]

with \( v = \infty \) if \( w_n < \infty \) for all \( n \).

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