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ON MAHLER'S CLASSIFICATION IN LAURENT SERIES FIELDS

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ABSTRACT. In 1932, K. Mahler introduced his famous classification for complex numbers in disjoint sets A, S, T, U [**9, 10**]. In 1978, P. Bundschuh introduced a similar classification for the field of formal Laurent series over a finite field K and gave some explicit series in the class U. Here we consider the case of an arbitrary field K and prove the existence of U-numbers whose continued fractions verify additional properties.

0. Introduction. T. Schneider's book [**16**, Chapter 3] is a complete introduction to the subject, whereas A. Baker's book [**2**, Chapter 8] offers a general outlook. For a polynomial $P = c_n x^n + \cdots + c_0$ in $\mathbf{Z}[X]$ with $c_n \neq 0$, we define the degree $d(P)$ and the height $h(P)$ by

$$d(P) = n, \qquad h(P) = \mathrm{Max}\,\{|c_j|, 0 \leq j \leq n\}.$$

For natural numbers $n \geq 1$, $H \geq 1$, we consider

$$P_{n,H} = \{P \in \mathbf{Z}[X] : d(P) \leq n, H(P) \leq H\} \tag{1}$$

and for any complex number ξ, we define $w(n, H, \xi)$, $w_n(\xi)$, $w(\xi)$ by

$$\mathrm{Min}\,\{|P(\xi)| : P \in P_{n,H}\} = H^{-nw(n,H,\xi)}$$

and

$$\begin{aligned} w_n = w_n(\xi) &= \limsup_{H\to\infty} w(n, H, \xi);\\ w = w(\xi) &= \limsup_{n\to\infty} w_n(\xi) \\ v = v(\xi) &= \mathrm{Inf}\,\{n : w_n(\xi) = \infty\} \end{aligned} \tag{2}$$
with $v = \infty$ if $w_n < \infty$ for all n.

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