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ESTIMATES FOR L-FUNCTIONS ASSOCIATED WITH SOME ELLIPTIC CURVES

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1. Introduction. Among the elliptic curves over $\mathbf{Q}$ written in the canonical form $y^2 = x^3+Ax+B$, only two families have automorphisms different from the identity and the hyperelliptic involution, namely (see [**1**, p. 93])

$$E : y^2 = x^3 - Dx \tag{1}$$

$$E : y^2 = x^3 + D. \tag{2}$$

All of these curves have complex multiplication, and it follows from the work of M. Deuring (see [**3**]) that they are modular. Consequently, there are various Dirichlet series naturally associated with them. In this note we examine the Hasse-Weil L-function and its symmetric square and obtain, by quite familiar means, estimates of their special values which are explicit with respect to the conductor. The analogue of these results for the Dirichlet L-functions is considered to be out of reach by current methods.

For simplicity we assume D is squarefree and $2, 3 \nmid D$. Hence the conductor, N, is a multiple of D^2 (see Appendix C of [**12**]).

In the next three sections we shall deal with the curve (1) and later we show how to modify the result for the curve (2). In the aforementioned sections, p and q will denote prime numbers satisfying $p \equiv 1 \pmod 4$ and $q \equiv 3 \pmod 4$.

2. The Hasse-Weil L-function and its symmetric square. The Hasse-Weil L-function of (1) is defined by (see [**5**, Chapter 18])

$$\begin{aligned} L(s) = L_E(s+1/2) &= \sum_n a(n) n^{-s} \\ &= \prod_{q\nmid N} (1+q^{-2s})^{-1} \prod_{p\nmid N} (1-a(p)p^{-s}+p^{-2s})^{-1}, \end{aligned}$$

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