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THE SEQUENCE x/n AND ITS SUBSEQUENCES

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1. Introduction. We begin by mentioning two problems which seem to have no relation to each other.

Problem 1. *A positive integer n is said to be sparsely totient if*

$$\phi(m) > \phi(n)$$

for all $m > n$, where ϕ is Euler's function. Find the smallest number λ such that, for all sparsely totient numbers n, we have

$$\max_{p|n} p = O_\varepsilon((\log n)^{\lambda+\varepsilon}). \tag{1.1}$$

Here and subsequently, p denotes a prime number and ε an arbitrary positive number.

Now let K be an algebraic number field with degree d; the *size* of an algebraic integer θ in K is the maximum of the set of absolute values of the d conjugates of θ. Let $\alpha_1, \dots, \alpha_n$ be $n \geq 3$ distinct algebraic integers in K and μ a nonzero algebraic integer in K.

Problem 2. *Give a bound for the size of solutions X, Y of the Thue equation*

$$(X - \alpha_1 Y) \cdots (X - \alpha_2 Y) = \mu$$

in algebraic integers X, Y.

Such a bound can be expressed in terms of d and the heights of $\alpha_1, \dots, \alpha_n$, μ and some algebraic integer generating K [**1**, Section 4.2].

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