ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 4, Fall 1996

WEAK SEQUENTIAL COMPLETENESS OF β-DUALS

CHRISTOPHER E. STUART

1. Introduction. The property of weak sequential completeness in sequence spaces has been considered by many authors and has been used to prove results in summability theory and functional analysis (see [**2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 14, 15**]). In this paper we present a generalization of the following theorem of D. Noll (see below for relevant definitions):

Theorem 1.1 [**9**, Theorem 6]. *If E is a sequence space containing Φ that has the weak gliding hump property, then E^β is $\sigma(E^\beta, E)$-sequentially complete.*

We show, in Theorem 3.5, that if E is a sequence space containing Φ that has the signed weak gliding hump property (Definition 3.4), then E^β is $\sigma(E^\beta, E)$-sequentially complete. The sequence space of bounded series, bs, is shown to have the signed weak gliding hump property. It is known that bs fails the weak gliding hump property (see [**9, 5**]).

2. Preliminaries. A *sequence space* is a vector space of sequences, which can be scalar ($\mathbf{R}$ or $\mathbf{C}$) or vector-valued. In this paper all vector spaces are over $\mathbf{R}$, largely for convenience.

A real-valued sequence space E is called a *K-space* if the inclusion map $E \to \omega$ (the space of all sequences) is continuous, when ω is given the product topology ($\omega = \prod_{i=1}^{\infty} (\mathbf{R})_i$). A K-space with a Frechet (complete, metrizable and locally convex) topology is called an *FK-space*; if the topology is a Banach topology, then E is called a *BK-space*.

The α-, β- and γ-duals of a sequence space E are defined to be

$$E^\alpha = \left\{ (y_i) : \sum_{i=1}^{\infty} |x, y_i| < \infty \text{ for all } (x_i) \in E \right\},$$

Received by the editors on August 12, 1993.

Copyright ©1996 Rocky Mountain Mathematics Consortium