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INTEGRATION AND L_2-APPROXIMATION: AVERAGE CASE SETTING WITH ISOTROPIC WIENER MEASURE FOR SMOOTH FUNCTIONS

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ABSTRACT. We propose isotropic probability measures defined on classes of smooth multivariate functions. These provide a natural extension of the classical isotropic Wiener measure to multivariate functions from C^{2r}. We show that, in the corresponding average case setting, the minimal errors of algorithms that use n function values are $\Theta(n^{-(d+4r+1)/(2d)})$ and $\Theta(n^{-(4r+1)/(2d)})$ for the integration and L_2-approximation problems, respectively. Here d is the number of variables of the corresponding class of functions. This means that the minimal average errors depend essentially on the number d of variables. In particular, for d large relative to r, the L_2-approximation problem is intractable. The integration and L_2-approximation problems have been recently studied with measures whose covariance kernels are tensor products. The results for these measures and for isotropic measures differ significantly.

1. Introduction. We study the integration and L_2-approximation problems for multivariate functions f. For the integration problem, we want to approximate the integral of f, and for the function approximation problem, we want to recover f with respect to the L_2-norm. For both problems, we want to determine methods with minimal error among all methods that use n function values. Moreover, we want to know how these errors depend on the number n of evaluations, on the number d of variables of f, and on regularity of f.

Both problems have been extensively studied in the literature, see, e.g., [**15, 24, 25**] for hundreds of references. However, they are mainly addressed in the worst case setting with the algorithm cost and error measured by the worst performance with respect to a given class F of functions. Depending on the smoothness properties of functions from

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