ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 4, Fall 1996

ASYMPTOTIC BEHAVIOR OF ORTHOGONAL RATIONAL FUNCTIONS CORRESPONDING TO MEASURE WITH DISCRETE PART OFF THE UNIT CIRCLE

K. PAN

ABSTRACT. For a positive measure μ on the unit circle in the complex plane, m points off the unit circle $z_1, \dots, z_m$ and m positive number A_j, $j = 1, 2, \dots, m$, we investigate the asymptotic behavior of orthogonal rational functions $\psi_n(z)$, $n = 0, 1, 2, \dots$, with prescribed poles lying outside the unit circle corresponding to $d\mu/2\pi + \sum_{j=1}^{m} A_j \delta_{z_j}$, where δ_z denotes the unit measure supported at point z. We find the relative asymptotics of $\psi_n(z)$ with respect to the orthogonal rational functions corresponding to $d\mu/2\pi$ off the unit circle.

1. Introduction. Let $d\mu$ be a finite positive Borel measure with an infinite set as its support on $[0, 2\pi)$. We define $L^2_{d\mu}$ to be the space of all functions $f(z)$ on the unit circle $T := \{z \in \mathbf{C} : |z| = 1\}$ satisfying $\int_0^{2\pi} |f(e^{i\theta})|^2 \, d\mu(\theta) < \infty$. Then $L^2_{d\mu}$ is a Hilbert space with inner product

$$\langle f, g \rangle := \frac{1}{2\pi} \int_0^{2\pi} f(e^{i\theta}) \overline{g(e^{i\theta})} \, d\mu(\theta).$$

We define $\mathcal{P}_n$ to be all polynomials with degree at most n. For any polynomial q_n with degree n, we define $q_n^*(z) = z^n \overline{q_n(1/\bar{z})}$. Consider an arbitrary infinite sequence $\mathbf{S} = \{\alpha_n\}$ with $n \in \mathbf{N}$ and $|\alpha_n| < 1$, and let

$$b_k(z) := \frac{\alpha_k - z}{1 - \bar{\alpha}_k z} \frac{|\alpha_k|}{\alpha_k}, \qquad k = 1, \dots ,$$

where for $\alpha_k = 0$ we put $|\alpha_k|/\alpha_k = -1$. Next we define finite Blaschke products recursively as

$$B_0(z) = 1 \quad \text{and} \quad B_k(z) = B_{k-1}(z) b_k(z), \qquad k = 1, \dots .$$

Received by the editors on August 25, 1994, and in revised form on January 25, 1995.