

DOMAINS OF TRIGONOMETRIC TRANSFORMS

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ABSTRACT. It is shown that the maximal solid domains of trigonometric transforms coincide with appropriate amalgam spaces.

1. Introduction. To formulate our objectives in this note, we need to recall the definition and some properties of the extended (maximal solid) domain of an integral operator. For details, we refer to [2] and [7].

Let X, Y be σ -finite measure spaces with the measures denoted by dx, dy , and let $L^\circ(X), L^\circ(Y)$ denote, as usual, the spaces of measurable finite a.e. (complex valued) functions on X and Y , respectively. L° is an F -space (vector metric complete) with the topology of convergence in measure on all subsets of finite measure. This topology on $L^\circ(x)$ may be defined, for instance, by any of the F -norms of the form $u \rightarrow \rho_x(u) := \int_x \Phi(|u(x)|)\phi(x) dx$, where Φ is a positive increasing continuous subadditive bounded function from \mathbf{R}_+ into $(0, 1)$ with $\Phi(0) = 0$ and $\phi > 0$ with $\int_\phi < \infty$. The F -norms ρ_Y are defined similarly. It is convenient to assume that some such F -norms ρ_X, ρ_Y have been fixed throughout this paper.

For a kernel k , a measurable function on $X \times Y$, the corresponding integral operator

$$K : D_K \subset L^\circ(Y) \rightarrow L^\circ(X)$$

is defined by:

(1.1)

$$Ku(x) = \int_Y k(x, y)u(y) dy,$$

$$D_K = \left\{ u \in L^\circ(Y); K_a u(x) = \int_Y |k(x, y)||u(y)| dy < \infty \text{ a.e.} \right\}.$$

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