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KRONROD EXTENSION OF GENERALIZED GAUSS-RADAU AND GAUSS-LOBATTO FORMULAE

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ABSTRACT. Kronrod extensions of Gauss-Radau and Gauss-Lobatto formulae having end points of multiplicity 2 are studied. For the four Chebyshev measures, expansions of the respective Stieltjes polynomials in terms of appropriate Chebyshev polynomials are given whenever possible; otherwise, an efficient computational algorithm is given. Explicit formulas are derived for the weights associated with the end points.

1. Introduction. In 1964, Kronrod [**9, 10**] initiated the idea of extending Gaussian quadrature formulae. He proposed to add $n+1$ nodes to an n-point Gauss-Legendre formula and to choose the new nodes and all weights of the extended formula so that it has maximum degree of exactness. It turns out that the additional nodes are the zeros of a polynomial of degree $n+1$ (known as Stieltjes polynomial) that is orthogonal to all lower-degree polynomials with respect to the Legendre polynomial of degree n as the weight function. Work in this direction has intensified in the last ten years. The reader is referred to surveys by Gautschi [**3**] and Monegato [**12**] for a detailed discussion of recent developments in this area. Here we apply Kronrod's idea to generalized Gauss-Radau and Gauss-Lobatto formulae with double end points, recently discussed by C. Bernardi and Y. Maday [**1**], and Gautschi and Li [**6, 7**].

We assume that the weight function w associated with the integration is one of the four Chebyshev weights:

$$\begin{aligned}
w_1(t) &= (1-t^2)^{-1/2},\\
w_2(t) &= (1-t^2)^{1/2},\\
w_3(t) &= (1-t)^{-1/2}(1+t)^{1/2},\\
w_4(t) &= (1-t)^{1/2}(1+t)^{-1/2}.
\end{aligned}$$

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